Calculation of distance travelled by an insect climbing the circular trunk of a tree:

Case 1: When the insect keeps climbing at a fixed inclination to the horizontal throughout and move around the tree trunk:

Let us consider a typical example.

Example: If an insect starting from the ground climbs a circular tree trunk of diameter 50 cm at a fixed inclination to the horizontal throughout and reaches an altitude of 150 cm from the ground after 3 complete revolutions around the trunk,

a) What is the total distance covered by the insect?

b) What is the fixed angle of inclination that it makes with the horizontal?

Solution:

a)

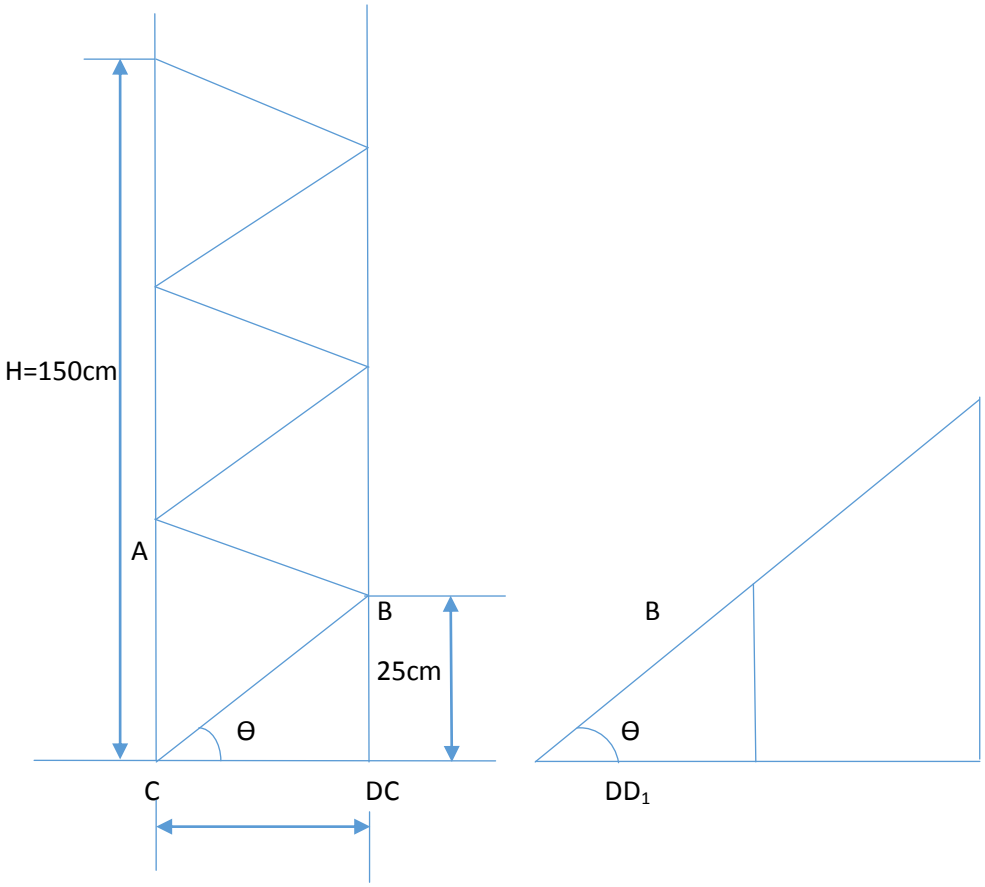

Diagram 1 (Projected path of the distance travelled by the insect)

Diagram 2 (Actual path travelled by insect)

For 3 revolutions, altitude reached by the insect=150 cm.

Hence, for half the revolution, it would reach 150/6=25 cm.

While reaching this distance, the insect is travelling at a fixed inclination.

The projected distance travelled by the insect is given the hypotenuse of the right triangle CBD in

diagram 1.

The actual distance travelled by the insect is given by hypotenuse of right triangle CB_1D_1 in diagram 2 (This is obtained by straightening the curved path travelled by the insect for half revolution around the trunk).

In right triangle CBD,

$CB^2 = CD^2 + BD^2$ (Phythagorous theorem).

Hence $CB = \sqrt{(CD^2 + BD^2} = 55.9$ cm.

As CBD and CB_1D_1 are similar triangles,

$CD/CD_1 = CB/CB_1$ ……(1).

But CD_1 is the line segment obtained by straightening the perfect semi-circle. Hence ,it's length equals $50*(\pi/2)$.

Substituting the values of CD_1 ,CB and CD in equation (1), we get

$CD_1 = 87.8075$ cm. This is for half the revolution.

As length travelled by the insect is exactly 6 times (1/2) the revolution's distance, it is (6*87.8075)=526.845 cm.

b) In triangle CBD, sin Θ = Opposite /hypotenuse= (25/55.9)

hence $\Theta = \sin^{-1}(25/55.9) = 26.566^\circ$.

Generalising with the above example, if H denotes the total altitude covered by the insect, H/(2*N) represents altitude covering half-the revolution,where N denotes the number of revolutions.

Hence distance travelled by the insect **$D= (2*N)* (\sqrt{(H^2/(4*N^2))}+d^2)*(\pi/2)$**

Where 'd' denotes the diameter of the circular trunk of the tree.